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ABSTRACT

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The Calculator's Effect on Children's Solution Processes

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Paper presented at the Fifty-seventh Annual Meeting of the National Council of Teachers of Mathematics, Boston, Massachusetts, April 18-21, 1979.

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SE 027 673

Abstract

Three preliminary studies and one exploratory main study investigated the effect of calculator use on elementary, middle, and secondary school students' solution methods on routine mathematical problems. Items were identified for which a longer and a shorter solution method existed. Half of the sample worked these using a calculator and half used paper and pencil.

For the 83 eighth-grade Ss in the main study there was a significant association between calculator use and choice of the longer solution method on 5 of 7 items. The trend in all three preliminary studies also supported this finding. The heuristics of simplification and application of a structure property such as distributivity appear to be calculator-inhibited rather than calculator-enhanced in the absence of specific instruction designed to encourage their use in conjunction with calculator computations.

In one of six classes Ss working with paper and pencil took significantly less solution time than those using the calculator. However, items were restricted in that they involved whole number computations only, and Ss using the calculator were required to record all keystrokes.

Performance in terms of the number of items correct was roughly comparable for the two groups (the Ss using the calculator had slightly more correct responses). The number of errors in conceptualization of the problem does not appear to be affected by calculator rather than paper and pencil use. There were more computational errors with paper and pencil, and more errors related to the chaining of operations with the calculator.

The authors conclude that calculator use should be accompanied by instruction designed to encourage calculator users to search for simplifications and shorter solution methods. Additional research attention should focus on the calculator's effect on the user's responses in various mathematical situations.

This paper reports the results of several pilot investigations and an exploratory study designed to examine the effects of calculator usage on children's solution processes as applied to selected mathematical problems. A secondary purpose of the studies was to examine the calculator's effect on several product measures, namely, solution time and number of correct responses.

Background

The decision to introduce hand-held calculators (HHC) into a classroom should not be considered lightly. The adoption of the HHC represents a "package deal," in that it involves both potential gains as well as potential losses. The gains, of course, must be maximized while potential losses must be minimized or avoided, if possible. If some loss is inevitable, our responsibility as educators is clear. We must be aware of the losses and insure that the gains outweigh them.

Most of the scholarly attention which the HHC has received has focused on the potential gains or losses associated with its use in traditional instructional situations, particularly with respect to students' attitudes and on standardized measures of mathematics achievement. These studies generally have been "global" in nature, focusing on comparisons between experimental and control groups as indexed by broad measures of mathematics achievement. Suydam (Note 1) summarizes much of this research, indicating the following:

> In most of the studies at the elementary school level, the data were collected to provide an answer (to parents and school boards, as well as to teachers) to the question, "Will the use of calculators hurt mathematical achievement?" The answer appears to be "No;" in all but a few studies, achievement is as high or higher when cal-

> culators are used for mathematics instruction (but not used on tests) than when they are not used. ...the calculator, in general, facilitates mathematical achievement across a wide variety of topics and this finding is verified at both the elementary and secondary school levels. (p. 7)

Although cries have resounded that "minds will rot" and that future generations will not be able to function mathematically without a HHC available, these claims have not been supported empirically. Good examples of investigations which have focused on possible detrimental effects of HHC usage on aspects of children's mathematical achievement have been conducted by Shumway (Note 2) and Moser (Note 3). In brief, these studies support the hypothesis that the HHC, if employed instructionally in certain ways, has no debilitating effects on some aspects of children's mathematical performance.

However, mathematical performance in the 'product' sense is but one area in which the HHC could have unfavorable effects. Another aspect of children's mathematical performance which could be affected has been identified by McClintock (Note 4). His concern is the effect of HHC usage in the "process dimension," calling this area of HHC-related research a vital one "which remains virtually untouched." McClintock notes the suggestion of the authors of the NACOME report (National Advisory Committee on Mathematics Education)(1975), Shumway (1976) and Suydam (1976) that "the calculator may have the potential to free a student's mind to focus on the reasoning processes and problem solving strategies" (p. 1). However, if some safeguards are not implemented, in some sense, the reverse may be true. Students may become little more than button pushers, giving little thought to organizing the problem data or consideration to more efficient solution methods.

Attention to the process dimension is but one aspect of the larger

issue of local rather than global effects of HHC use. Local effects can be addressed empirically in two ways. One method focuses on instruction and examines the feasibility and effectiveness of using calculator-based or calculator-assisted instruction (see Weaver, 1976) for teaching certain mathematical concepts and generalizations. Professor J. Fred Weaver's on-going explorations at the Wisconsin Research and Development Center for Individualized Schooling and at the University of Wisconsin-Madison, (for example, Weaver, Note 5) and that of several federally funded projects (see Suydam, Note 1) have explored the effect of the HHC on promoting the learning of specific mathematical content.

A second approach to assessing local effects of HHC use centers on the learner and examines the HHC's effect on the user's mathematical behaviors exhibited in specific problem situations. These effects might be positive or negative. Examples of positive effects include improved estimation skills (producing a good estimation of final and intermediate solution results); efficient use of "guess-and-test" strategies; use of iteration, interpolation, or extrapolation skills; and increased use of problem-solving heuristics such as simplification or application of a structure property such as distributivity of multiplication over addition in search of a more efficient or direct solution. Possible negative effects include inappropriate or inefficient use of "guess-and-test" strategies or the aforementioned inattention to problem organization and to alternative solution methods.

The investigations reported in the body of this manuscript address the issue of local effects of HHC use by focusing on the user's solution method and success in dealing with selected mathematical problems. It was

anticipated that these studies could begin to contribute to the identification of a class of heuristics which are calculator-enhanced or calculator-inhibited.

The investigations evolved from a HHC-related research project conducted by Professor Weaver. In one classroom session, an item similar to the following one was presented to an accelerated class of seventh-grade pre-algebra students who had been working with an RPN-based HHC for several months:

> 28 students chartered a bus to attend a ball game. The total cost of the bus was $35.00. How much money must each student contribute?

If one gives some preliminary consideration to this problem, it should be apparent that each student must contribute at least one dollar. The division of the problem data, in the existing but wrong order (28 ÷ 35), produces the HHC display of .80, or 80 cents per student. Much to the surprise of educators involved in this session, this response was by far the most popular one. Questions were raised as to whether the results would have been similar if students were working with paper and pencil or if they used other than an RPN-based calculator and if this were not the case, what other kinds of mathematical problems would produce differences in performance.

Research Design and Procedure

Sequence of Investigations

Three preliminary investigations were conducted in schools in which only a single class had been using the HHC as part of their mathematics instruction. In these investigations, the inventories described below were varied in an attempt to identify items on which different (longer or shorter) solution methods were used. The final study used selected items

from the three preliminary studies. It was conducted in a school in which three classes were available which had used the calculator as part of their mathematics instruction.

Subjects

The sample across the four studies consisted of 152 subjects from intact classes at a variety of grade levels from elementary, junior high, and high school. Classes were chosen for their availability with respect to the criterion of having used the HHC as part of their regular instruction. Table 1 summarizes the relevant characteristics of the subjects.

Insert Table 1 about here.

Note that the 83 subjects in the three average-ability eighth grade classes comprised the sample for the main study. Forty-two of these subjects used the HHC and forty-one used paper and pencil (P&P).

All subjects had previous classroom HHC experience ranging from occasional use (approximately once per 10-15 classes) to constant availability and frequent use. In each case, the HHC was not used simply as an available computational aid. The calculators which had been used for instruction were the ones which subjects used in the studies.

Inventories

Construction

In all four studies, each item presented to subjects was one of two distinct types. In one case, the problem could be solved using one or more solution methods. One solution method involved more steps or computations than a second possible solution method. The problem itself may even 'suggest' use of the longer approach because of the way it is presented to the subjects. Problems 1 and 2 in Table 2 are examples of this

problem type. The first of these items involves the concept of distributivity of multiplication over addition, while the second item involves 'doing-undoing' or the inverse relationship between addition and subtraction. In each case, two possible solution methods are presented below the problem. As a parenthetical note, throughout the course of this investigation, at no time did a subject employ a successful solution method which was not one of the two methods identified by the authors.

Insert Table 2 about here.

In the second problem type, what is of interest is children's success or failure on the item in question. Items 3 and 4 in Table 2 are relevant examples. In this case, if the user simply keys data into the HHC in left-to-right order as it appears in the item, then an incorrect solution will be obtained. As a second parenthetical note, the reader should be aware that there are at least four distinct calculator languages (or logic systems) and the correct entry into the HHC of problem data may vary depending upon the language of the user's HHC. For a more detailed analysis of HHC languages and the implications for instruction, see Mitchell & Blume (Note 6).

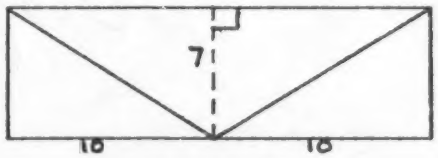
The complete inventories from the four investigations are given in Appendix A. In addition to the Distributivity and Doing-Undoing examples in Table 2, examples of other Type One items used in the four studies are given below:

Distributivity (symbolic form): Solve for N: $(16 \times 85) + (6 \times 85) = N$

"Cancellation:" Solve for N: $\frac{15 \times 161}{23 \times 15} = N$

"Disguised" distributivity: 16 cases of records with 108 records per case were delivered to the storeroom. There were already 151 loose records piled in the storeroom. During the week, 14 cases of 108 records were sold. How many records were left in the storeroom at the end of the week?

An area problem: Find the area of the rectangle.



Commutativity, Associativity: Solve for N: 4 x 89 x 25 = N

Multi-step: The star player on the basketball team scored 333 points during the first 9 games. For each game Joe averaged 19 points less than the star. How many total points did Joe score during the first 9 games?

In addition to the Order of Operations problem in Table 2, the other Type Two problem was a division problem similar to that on page 3. Type Two problems were originally included to provide a contrast to problems which had more than one solution method.

In the preliminary studies in which each subject completed two inventories, items were equated for problem structure and randomly assigned to one of the inventories (HHC or P&P). Items which did not discriminate in terms of solution methods were dropped from subsequent studies and several items were eliminated for the 5th-6th grade subjects due to difficulty level. In the main study, the HHC and P&P inventories were identical except for the directions.

It should be noted that the items on all inventories were "routine" problems in that students had available algorithmic procedures for their solution. These items were not designed to be "problems" in the sense of those in much of the mathematical problem-solving literature. Constants

and answers were restricted to the whole number domain to avoid increasing the P&P computational difficulty level.

Administration

In all cases, inventories were randomly distributed to classes of subjects. The inventory directed them to work with P&P or with the HHC. If working with the HHC subjects were instructed to use the HHC wherever they would normally use P&P to compute. Subjects were instructed to write down the sequence of keys which were pushed in working a problem. Computations performed mentally were not to be included in this sequence. Subjects working with P&P were directed to show their work, again with the exception of mental computations.

For the three preliminary investigations, each subject completed two inventories, one with HHC and one with P&P. As the first inventory was completed, the second was given to the subject and the HHC was then either provided or removed.

In all cases, the order in which subjects completed the inventory was recorded to provide rank-order data for comparing the solution time required by subjects using HHC to that of subjects using P&P.

Scoring

Coding was done by the authors independently and results were compared. No disagreements existed, although data on several problems for two subjects was eliminated due to their failure to follow directions.

Type One problems were scored for correctness, for the type of solution method, and for the type of error, if applicable. Coding categories were:

Answer: Correct; Incorrect

Method: Longer method; Shorter method; Other conceptually incorrect

method or no attempt.

Errors: Computation, copying or keystroke error; Order of operations error; Misconceptualization of the problem; No attempt.

Type Two items were scored as above with the exception that the method category was inapplicable.

Data Analysis

In the early stages of this research, two statistical designs were considered. In the preliminary studies, students first worked with HHC (or P&P) and then switched to the other on a second inventory. Approximately half of the subjects would start with the HHC and the rest with P&P. However, this design was replaced in the main study by what was considered to be a superior design. In the latter case, subjects were simply randomly assigned to a P&P or a HHC inventory. The data collected from these two groups were then compared. This approach avoids the repeated measures situation and the concern that subjects would remember how they solved similar items from the earlier inventory. This second approach also has the effect of greatly increasing sample size, since many of the subjects in the first situation generated non-useful data, i.e., a subject whose solution method did not change with the immediate switch from HHC to P&P did not contribute to the change cells in the first design.

Data were analyzed only from the first inventories in the preliminary studies. The data on change in solution methods within subjects across inventories from the 11-12th grade study is included in Appendix B but was not further analyzed. The association between HHC or P&P use and choice of longer or shorter solution methods was analyzed for each item by computing a phi-coefficient. Significance testing was done at the .05 level.

Data solution time (rank order of completion of the inventory) for

subjects in each class was analyzed for difference between HHC and P&P groups using the Kruskal-Wallis test.

The number of correct responses was recorded separately for each item of the main study for the HHC and P&P groups in each of the four studies.

Results

Main Study

Table 3 gives the solution method frequencies for each problem for the HHC and P&P subjects. Since the subjects who had used conceptually incorrect methods did not contribute useful information to this analysis, they were not included in the subtables of Table 4. Each of these subtables reports the number of subjects using longer or shorter methods for each of the items of Type One. Each of the subtables of Table 4 was tested with a phi-coefficient for the presence of significant ($\alpha \leq .05$) association between the type of aid used (HHC or P&P) and the type of solution method used (longer or shorter). There was no significant association on the verbal format Distributivity item and there was no use of the shorter method on the symbolic format Distributivity item. On each of the other five items, there was a significant association of the shorter method with P&P use and the longer method with HHC use. Across the seven problems of Type One, 23% of P&P responses (which used the longer or shorter methods) were of the shorter type while only 10% of the HHC responses were of the shorter form.

Insert Tables 3 and 4 about here.

On the Type Two (order of operations) item, all 42 HHC subjects worked the item incorrectly, and only 2 of the 41 P&P subjects worked this item

correctly.

Preliminary Studies

In the first (7th grade) preliminary study, performance across the two inventories completed by each subject indicated a slight preference on several items for the shorter method when P&P was used. This data is given in Table 5.

Insert Table 5 about here.

In the second preliminary study (11-12th grade) there were differences in use of longer and shorter methods for 5 of 6 items of Type One. On the other item (the Commutativity, Associativity item) no subject chose the shorter method. Table 6 reports the use of longer and shorter methods on the first test for the five items. None of the phi-coefficients was significant, although in all five instances, the shorter method was preferred more often by the P&P group. Across the five items, the P&P group used the shorter method 34% of the time while the HHC group did so only 6% of the time.

Insert Table 6 about here.

In the third (5-6th grade) preliminary study, there were again slight differences in the preference for longer and shorter methods. Table 7 gives frequencies of use of the two methods on the first test for three items of Type One. None of the phi-coefficients was significant, but the preference for shorter method was again more frequent in the P&P group.

Insert Table 7 about here.

The above discussion is concerned only with items which were used in the main study. On other items, differences were too small (or non-existent) to warrant their inclusion in further studies. The pattern of longer and shorter method use in the preliminary studies mirrors the association between P&P use and shorter method found with the same items in the main study. Table 8 compares the differences in use of longer and shorter methods across the four studies for the HHC and P&P groups.

Insert Table 8 about here.

The Type Two problem in the 11-12th grade study was completed correctly by 50% (5 Ss) of the P&P group and by 80% of the HHC group. It is interesting to note that several subjects did not complete this item correctly even though they were using a calculator with the standard order of operations hierarchy built in. In the 5-6th grade sample, only 2 of the subjects completed the order of operations item correctly.

Solution Time

Results regarding the amount of time necessary for completion of the inventories are given in Table 9. The Kruskal-Wallis test was significant ($p < .05$) in only one of three classes in the main study, indicating that the P&P subjects finished sooner than the HHC group. In all other instances, (main and preliminary studies) the Kruskal-Wallis test indicated no significant differences in the order in which the two groups completed the inventory. It should be noted that in all cases, the existing differences in rank sums indicated that the P&P group used less time to complete the inventory than the HHC group.

Insert Table 9 about here.

Number of Correct Responses

The number of correct responses to each of the items in the main study is given in Table 10. While overall performance is slightly better for the HHC groups, the pattern of differences is mixed across items. The poorer performance of the HHC group on the Disguised distributivity item and the Distributivity (symbolic format) item can in part be traced to errors in the chaining of operations on the HHC. For example, to compute (16 x 85) + (6 x 85), one must record products and then add, unless a calculator is available which follows the order of operations hierarchy. More than 25% of the subjects in the HHC group (main study) used the incorrect keystroke sequence: 16, x, 85, +, 6, x, 85, = on this item.

Insert Table 10 about here.

Types of Errors

The four types of errors were tabulated for each problem in the main study and appear in Table 11. Errors in category E_2 include errors in which operations were improperly chained in calculators without an order of operations hierarchy.

Insert Table 11 about here.

Discussion

The results of the main study and the supporting evidence from the three preliminary studies indicate that across a variety of grade levels, choice of shorter solution methods was associated with P&P rather than HHC use. The authors hypothesize that this difference was due to the HHC subjects' lack of attention to organization of the problem data or lack of any search for alternative solution methods. The simplification and application of structure property heuristics appear to be calculator-inhibited rather than calculator-enhanced.

Although the use of shorter solution processes for the HHC and P&P groups were on the order of 10% and 25%, respectively, across the studies, this difference seems large enough to warrant attention in the design of instruction which incorporates use of the HHC. The investigations reported here are not to be taken as an indictment of HHC use in the classroom. Rather, they are to be interpreted in light of suggestions for instruction. If it is mathematically desirable that students become accustomed to thinking before they compute, then it becomes imperative that teachers devote sufficient attention to alerting students who are using the HHC that prior thought, consideration of alternatives and attention to potential simplification of the problem solution process must occur when using the HHC as well as when using P&P. Left to their own, students may systematically overlook desirable simplifications and the search for shorter solution processes. It seems likely that this potential negative effect of HHC use can be avoided, however, by encouraging students to give prior thought to problems regardless of the availability of the HHC. Continued emphasis in this area during classroom use of the HHC (especially when used by the teacher) may be sufficient to minimize or avoid any propensity to fail to apply some of the student's available and useful problem solving heuristics.

Given the types of items used in this study, it appears that use of the HHC does not reduce the time necessary for solution. This statement must be qualified by the fact that students using the HHC were required to record all keystrokes in addition to recording the result. On problems which involve two and three-digit whole number constants and whole number answers, these studies indicate that students working with P&P can complete items as quickly as students who use the HHC and record keystrokes. This seems only to say that one should not expect savings in time with HHC usage unless the HHC is used for problems on which it clearly provides an advantage over P&P.

Across all items in Table 10, the percentage of correct responses were approximately 69% with HHC and 64% with P&P. There is no striking difference in overall performance in terms of correct responses, although there are differences on specific items. An analysis of the error types in Table 11 indicates that there are more computational type errors with P&P, but that much of the computational advantage of the HHC in terms of accuracy is lost to errors caused by difficulties with the chaining of operations on simple algebraic logic calculators. It does not appear that the number of conceptual errors (in organization of the problem data and in choosing the operations to be performed) is affected by HHC rather than P&P use.

It should be noted that the above finding along with the uniformly poor performance on Type Two items (order of operations) suggests that educators take a careful look at the type of calculator which students are asked to use. Despite specific instruction on order of operations in the 5-6th grade and 8th grade classes, only 4 of 111 subjects correctly answered the Type Two item. Even when using the SR-40 calculator, the 11-12th grade trigonometry students failed to correctly complete the Type Two item in 5 of 20 instances. Perhaps the uniform use of RPN calculators might remedy this

difficulty to some extent. With the RPN logic system, the user is constantly forced to make the decision as to which operation is to be performed next. Perhaps more importantly, there is no need or reason to change from "non-hierarchy" to "hierarchy" HHC's when order of operations becomes a salient mathematical topic. The issue of which logic system is most advantageous for given mathematical objectives is one which deserves greater attention by researchers.

Limitations and Future Studies

One of the limitations of this investigation is its design. Optimally, if two samples are to be employed, the subjects should be as similar as possible in every respect with the exception of HHC experience. The reason for this is that subjects using the HHC in their classrooms or frequently at home may develop poor (or lazy) habits that may carry over to situations in which P&P are employed. As noted, differences observed in this manuscript were between 10% and 25% favoring P&P users. These differences may be much more profound when comparing subjects with HHC experience to subjects with little or no experience. Future studies should also begin paying much closer attention to the amount of HHC experience of the subjects employed in the investigation.

A second limitation is the relatively small number of item types included on the inventories. It is important that information be obtained on a much wider range of mathematical problems. In addition to the routine problems used in these investigations, it is important that the HHC's effect on the user's solution processes be investigated in the (non-routine) problem-solving setting.

The studies reported above provide evidence that there are some potential drawbacks associated with the use of the HHC without the support of

instruction designed to minimize or eliminate those drawbacks. One can only speculate as to the effectiveness of instruction designed for this purpose, but it seems reasonable that such instruction can be successful in circumventing any negative effects of HHC use.

In a limited sense, the heuristic of simplification and search for a shorter solution method is calculator-inhibited rather than calculator-enhanced. Given this result, the task remains to more exhaustively seek to identify other positive and negative effects of HHC use on the user's solution processes. If these can be identified and instruction tailored to facilitate or suppress their appearance, then mathematics educators will have taken an important step toward maximizing the instructional potential of the HHC.

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	Preliminary Studies 1	2	3	Main Study	
Grade Level	7th Accelerated	11-12th Trig	5-6th Mixed	8th Average	
# Classes	1	1	1	3	6
# Subjects	21	20	28	83 42-HHC 41-P&P	152
Classroom Experience	1 Year	1 Semester	1 Semester	2 Weeks	
Using Calculators	1 Day/Wk.	5 Days/Wk.	Occasional	5 Days/Wk.	
Calculator Used	Novus 4550 RPN	TI SR-40 Algebraic with Hierarchy	TI 1255 Algebraic	Littronix 2250 TI 1255 Algebraic	

Table 1. DESCRIPTION OF SUBJECTS

TABLE 2. ITEM AND SOLUTION EXAMPLARS

PROBLEM TYPE ONE

1. THE GIRLS' SOFTBALL LEAGUE HAS 22 TEAMS WITH 13 PLAYERS ON EACH TEAM. THE BOYS' SOFTBALL LEAGUE HAS 18 TEAMS WITH 13 PLAYERS ON EACH TEAM. ALTOGETHER, HOW MANY BOYS AND GIRLS PLAY SOFTBALL?
   A. $(22 \times 13) + (18 \times 13) = N$
   B. $(22 + 18) \times 13 = N$
2. $963 + 498 - 963 = N$
   A. FIRST ADD 963 AND 498, AND THEN SUBTRACT 963.
   B. NOTE THAT 963 IS ADDED AND LATER SUBTRACTED. RESPOND 498 WITHOUT COMPUTATIONS.
3. $27 + 12 \times 18 = N$
   A. ENTER THE DATA INTO THE HHC IN LEFT-TO-RIGHT ORDER.**
   B. FOLLOW ACCEPTED ORDER OF OPERATIONS AND MULTIPLY 12 BY 18 FIRST.
4. THERE ARE 36 CANS ON THE SHELF. 8 CASES WITH 12 CANS IN EACH WERE PLACED ON THE SHELF. HOW MANY CANS ARE NOW ON THE SHELF?
   A. $(36 + 8) \times 12 = N$
   B. $(8 \times 12) + 36 = N$

**SOME CALCULATORS HAVE A BUILD-IN HIERARCHY WHICH WOULD PRODUCE THE CORRECT SOLUTION HERE IF THE DATA WERE ENTERED IN THIS MANNER. HOWEVER, THESE HHC'S WERE ONLY USED IN THIS INVESTIGATION WITH THE 11-12TH GRADE SAMPLE.

		Distributivity verbal format	Disguised Distributivity	Distributivity (symbolic format)	Doing-Undoing (verbal format)	Doing-Undoing (symbolic format)	Cancellation	Multi-step
	L	31	28	42	38	36	39	20
HHC	S	5	8	0	2	4	1	7
(N=42*)	0	4	5	0	0	0	1	15
	L	29	18	40	31	28	34	13
P&P	S	1	16	0	9	12	6	15
(N=41)	0	11	7	1	1	1	1	13

Table 3. Solution Method Frequencies (Main Study)

L = Longer Method

S = Shorter Method

0 = Other (incorrect) Method or no Attempt

*Data for 2 subjects was not included for some problems

TABLE 4. USE OF LONGER AND SHORTER METHODS BY ITEM

A) DISTRIBUTIVITY (VERBAL FORMAT)

	HHC	P&P
L	31	29
S	5	1

ASSOCIATION (PHI-COEFFICIENT): .183 N.S.

B) DISGUISED DISTRIBUTIVITY

	HHC	P&P
L	28	18
S	8	16

-.262 SIG., ($P < .05$)

C) DISTRIBUTIVITY (SYMBOLIC FORMAT)

	HHC	P&P
L	42	41
S	0	0

(NO DIFFERENCE)

D) DOING-UNDOING (VERBAL FORMAT)

	HHC	P&P
L	38	31
S	2	9

-.254 SIG., ($P < .05$)

E) DOING-UNDOING (SYMBOLIC FORMAT)

	HHC	P&P
L	36	28
S	4	12

-.250 SIG., ($P < .05$)

TABLE 4. (CONT'D)

F) CANCELLATION

	HHC	P&P
L	39	34
S	1	6

-.221 SIG., ($P < .05$)

G) MULTI-STEP

	HHC	P&P
L	20	13
S	7	15

-.282 SIG., ($P < .05$)

TABLE 5. USE OF LONGER AND SHORTER METHODS ACROSS TWO INVENTORIES (PRELIMINARY STUDY 1)

DISTRIBUTIVITY (VERBAL)

	HHC	P&P
L	16	11
S	4	9

MULTI-STEP

	HHC	P&P
L	13	12
S	5	7

TABLE 6. USE OF LONGER AND SHORTER METHODS
(PRELIMINARY STUDY 2)

		HHC	P&P	ASSOCIATION (PHI-COEFFICIENT)
A) DISTRIBUTIVITY (VERBAL)				
	L	10	7	-.42 N.S.
	S	0	3	
B) DISGUISED DISTRIBUTIVITY				
	L	8	6	-.218 N.S.
	S	2	4	
C) DOING-UNDOING (SYMBOLIC)				
	L	9	5	-.436 N.S.
	S	1	5	
D) CANCELLATION				
	L	10	7	-.420 N.S.
	S	0	3	
E) MULTI-STEP				
	L	10	8	-.333 N.S.
	S	0	2	

TABLE 7. USE OF LONGER AND SHORTER METHODS (PRELIMINARY STUDY 3)

A) DISTRIBUTIVITY (VERBAL)

	HHC	P&P	
L	13	7	-.380 N.S.
S	0	2	

B) DISTRIBUTIVITY (VERBAL)

	HHC	P&P	
L	12	10	-.223 N.S.
S	0	1	

C) DOING-UNDOING (SYMBOLIC)

	HHC	P&P	
L	11	10	-.082 N.S.
S	3	4	

TABLE 8. COMPARISON OF RESULTS ON TYPE ONE ITEMS ACROSS FOUR STUDIES

	DISTRIBUTIVITY (VERBAL)	DOING-UNDOING	DISGUISED DISTRIBUTIVITY	CANCELLATION	MULTI-STEP	DISTRIBUTIVITY (SYMBOLIC)
MAIN STUDY (8TH GRADE)	N.S. --HHC	SIG. --P&P	SIG. --P&P	SIG. --P&P	SIG. --P&P	N.S. NO DIFFERENCE
PRELIMINARY STUDY 3 (5-6TH GRADE)	N.S. --P&P	N.S. --P&P	-	-	-	-
PRELIMINARY STUDY 2 (11-12TH GRADE)	N.S. --P&P	N.S. --P&P	N.S. --P&P	N.S. --P&P	N.S. --P&P	-
PRELIMINARY STUDY 1 (7TH GRADE)	--P&P	-	-	-	--P&P	-

N.S. = NOT SIGNIFICANT (AT OR BELOW THE .05 LEVEL)

--HHC = SHORTER METHOD FAVORED MORE OFTEN BY HHC GROUP THAN BY P&P GROUP

--P&P = SHORTER METHOD FAVORED MORE OFTEN BY P&P GROUP THAN BY HHC GROUP

TABLE 9. DIFFERENCES IN TIME NECESSARY FOR COMPLETION OF THE INVENTORY

STUDY	DIFFERENCES BETWEEN HHC AND P&P RANKS (KRUSKAL-WALLIS)	GROUP WITH SMALLER TOTAL RANK SUM FOR ORDER OF COMPLETION OF INVENTORY
7TH GRADE	N.S.	P&P
11-12TH GRADE	N.S.	P&P
5-6TH GRADE	N.S.	P&P
8TH GRADE (MAIN)		
CLASS 1	* ($p < .05$)	P&P
CLASS 2	N.S.	P&P
CLASS 3	N.S.	P&P

TABLE 10. NUMBER OF CORRECT RESPONSES (AS A FRACTION OF THE TOTAL)

		DISTRIBUTIVITY (VERBAL)	DISGUISED DISTRIBUTIVITY	DISTRIBUTIVITY (SYMBOLIC)	DOING-UNDOING (VERBAL)	DOING-UNDOING (SYMBOLIC)	CANCELLATION	MULTI-STEP	ORDER OF OPERATIONS
MAIN STUDY (8TH GRADE)	HHC	32/42	25/42	30/42	41/42	41/42	35/42	17/42	0/42
	P&P	27/41	29/41	35/41	33/41	35/41	31/41	22/41	2/41
PRELIM.3 (5-6TH)	HHC	22/28	-*	-	-	12/14	-	-	2/14
	P&P	11/28	-	-	-	12/14	-	-	0/14
PRELIM.2 (11-12TH)	HHC	10/10	10/10	-	-	10/10	8/10	8/10	8/10
	P&P	6/10	7/10	-	-	9/10	8/10	8/10	8/10
PRELIM.1 (7TH)	HHC	9/10	-	-	-	-	-	6/10	-
	P&P	10/11	-	-	-	-	-	7/11	-

* INDICATES THIS ITEM DID NOT APPEAR

TABLE 11. TYPES OF ERRORS BY ITEMS (MAIN STUDY)

		Distributivity (verbal)	Doing-Undoing (verbal)	Order of operations	Disguised Distributivity	Cancellation	Doing-Undoing (symbolic)	Multi-step	Distributivity (symbolic)
P&P	E_1	3	7	5	6	8	5	4	4
	E_2	0	0	37	0	0	0	0	0
	E_3	10	1	2	6	1	1	17	1
	E_4	1	0	0	1	1	0	1	1
HHC	E_1	3	1	0	3	0	1	2	1
	E_2	1	0	41	3	4	0	0	11
	E_3	6	0	1	9	2	0	23	0
	E_4	0	0	1	2	1	0	0	0

E_1 - COMPUTATION, KEYSTROKE, COPYING ERROR

E_2 - ORDER OF OPERATIONS - RELATED ERROR

E_3 - CONCEPTUAL ERROR

E_4 - NO ATTEMPT, UNDETERMINED

Appendix A

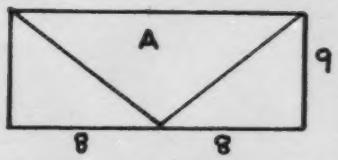
Inventories from the Four Studies

Preliminary Study 1: 7th grade (accelerated)

HHC test (half the sample worked this as their first inventory; half worked this following the P&P inventory)

Show your work on the recording sheets.

1. Find the area of triangle A which is inside the rectangle.



2. The star player on the basketball team scored 333 points during the first 9 games. For each game, Joe averaged 19 points less than the star. How many points did Joe score during the first 9 games?

3. Evaluate $\frac{2415}{15 \times 23}$.

4. John bought 17 bicycles at $15.67 each. After cleaning them up, he was able to sell them for $22.39 each. How much money did he make?

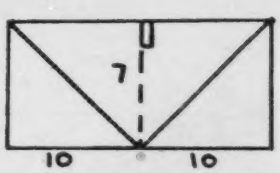
5. 28 players need $35 for equipment. How much must each one pay for the equipment?

P&P Test

Use no calculators. Show your work on the paper provided.

1. Evaluate $\frac{1632}{17 \times 24}$.

2. Find the area of the rectangle below.



3. Sally and her little brother Bill sell paintings that they make. Sally sold 13 of her paintings for $31.29 each, while Bill only received $27.85 for each of the 13 paintings that he sold. How much more money did Sally receive for her paintings than Bill?

4. The Badgers scored 184 points during their first 8 games. Their opponents averaged 12 fewer points per game. How many points did the

Badgers' opponents score against them in the first 8 games?

5. It will cost $36 to charter a bus to a Brewers game. If 45 people share the cost, how much will it cost each person?

Preliminary Study 2: 11-12th grade (trigonometry students)

HHC test (half the sample worked this as their first inventory; half worked this following the P&P inventory)

A calculator is available for these problems.

Use the calculator wherever you would normally use paper and pencil to compute, and show any keystrokes you use in the blank spaces below each problem.

1. Solve for N: $345 + 8 \times 23 = N$

2. A VW-Rabbit and a Mark IV each were given 9 gallons of gas. The VW-Rabbit traveled 288 miles on the 9 gallons before running out of gas. The Mark IV averaged 18 miles per gallon less than the VW-Rabbit for its 9 gallons. How many miles did the Mark IV travel before running out of gas?

3. The main floor of a movie theater has 33 rows of seats with 16 seats in each row. The balcony of the theater has 17 rows of seats with 16 seats in each row. How many seats are there in the whole theater?

4. Solve for N: $987 + 762 - 987 = N$

5. 17 cases of records with 108 records per case were delivered to the storeroom. There were already 154 loose records piled in the storeroom. During the week, 15 cases of 108 records were sold. How many records were left in the storeroom at the end of the week?

6. Solve for N: $4 \times 89 \times 25 = N$

7. Solve for N: $616 + 457 - 249 = N$

8. Solve for N: $\frac{17 \times 232}{29 \times 17} = N$

P&P test

Solve these problems without a calculator.

Any written computations you do should be left in the blank space below the problem.

1. Martha bought 18 used bicycles at $37.93 each. After fixing them, she was able to sell them for $66.56 each. How much total profit did she make on this shipment of bicycles?

2. Solve for N: $\frac{15 \times 161}{23 \times 15} = N$

3. Solve for N: $572 + 466 - 277 = N$

4. Solve for N: $225 + 389 - 75 = N$

5. Solve for N: $336 + 7 \times 19 = N$

6. Adrian made 17 deposits of $112 each into her checking account. Before these deposits, she had had $115 in the checking account. Later in the month, she wrote 13 checks, each for $112 out of her checking account. How much money was left in her account after writing these checks?

7. Solve for N: $97 \times 45 + 97 = N$

8. The star player on the basketball team scored 306 points during the first 9 games. For each game, Joe averaged 16 points less than the star. How many total points did Joe score during the first 9 games?

Preliminary Study 3: 5-6th grade

HHC test (half the sample worked this as their first inventory; half worked this following the P&P inventory)

Directions were identical to those of study 2

1. There are 16 fifth grade classes with 31 students in each class. The sixth grade classes also have 31 students in each class but there are only 14 sixth grade classes. Altogether, how many fifth and sixth graders are there?

2. Find the number that goes in the blank.

   $9 \times 24 + 9 =$ ____

3. Sharon started with 53 stamps in her collection. With the $18 she got for Christmas, she bought 17 packets of stamps that had 12 stamps in each packet. How many stamps does she have now?

4. The girls' softball league has 22 teams with 13 players on each team. The boys' softball league has 18 teams with 13 players on each team. Altogether, how many boys and girls are softball players?

5. Find the number that goes in the blank.

   $16 + 8 \times 14 =$ ____

Main Study: 8th grade

HHC test

Directions were identical to those of study 2.

*1. There are 24 girls' softball teams with 13 players on each team. There are 16 boys' softball teams with 13 players on each team. Altogether, how many boys and girls are softball players?

2. Sarah takes the number 32 and multiplies it by 17. She then takes her answer and divides it by 32. What is her final answer?

3. Solve for N: $176 + 9 \times 13 = N$

4. 16 cases of records with 108 records per case were delivered to the storeroom. There were already 151 loose records piled in the storeroom. During the week, 14 cases of 108 records were sold. How many records were left in the storeroom at the end of the week?

5. Solve for N: $\frac{15 \times 161}{23 \times 15} = N$

6. Solve for N: $763 + 935 - 763 = N$

7. The star player on the basketball team scored 333 points during the first 9 games. For each game Joe averaged 19 points less than the star. How many total points did Joe score during the first 9 games?

8. Solve for N: $(16 \times 85) + (6 \times 85) = N$

*The Distributivity item used in the main study was given in three formats indicating different levels of suggestivity of the use of distributivity. The two formats used in addition to that of problem 1 above were:

> There are 24 girls' softball teams with 13 players on each team. The boys' teams also have 13 players on each team but there are only 16 boys' teams. Altogether, how many boys and girls are softball players?

and

> There are 24 girls' softball teams. There are 16 boys' softball teams. There are 13 players on every team. Altogether, how many boys and girls are softball players?

Note that the third format is the most suggestive of the use of a sum followed by a product rather than the sum of two products.

These formats were balanced for frequency of occurance across the HHC and P&P inventories and were randomly ordered within each triple of the HHC and P&P inventories for distribution to subjects. It should be noted that the only occurrence of use of the shorter method on this item by either HHC or P&P users occurred with the latter format described above.

P&P test

Directions were identical to those of study 2.
Items were identical to those of the main study HHC test.

Distributivity

HHC \ P&P	L	S
L	16	4
S	0	0

Disguised Distributivity

HHC \ P&P	L	S
L	10	6
S	2	2

Doing-Undoing

HHC \ P&P	L	S
L	11	6
S	2	1

Cancellation

HHC \ P&P	L	S
L	14	3
S	1	2

Commutative, Associative

HHC \ P&P	L	S
L	20	0
S	0	0

Multi-step

HHC \ P&P	L	S
L	18	0
S	0	2

L = longer method
S = Shorter method

Appendix B. Change in solution method with change in aid used (P&P, HHC) for 11-12th grade subjects (Preliminary Study 2).